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Aircraft Height Estimation using 2-D Radar

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ABSTRACT

A method to infer height information from an aircraft tracked with a single 2-D search radar is presented. The method assumes level flight in the target aircraft and a good estimate of the speed of the aircraft. The method yields good results for medium to high altitudes, though performs weaker at low altitudes. The method can distinguish between high and low targets on a normal 2-D radar, and can reach a height resolution of 100 m provided the 2-D radar is optimised to the task.

Keywords: Radar tracking, aircraft height estimation, 2-D search radar, target tracking, aircraft altitude estimation, 2-D radar

1. INTRODUCTION

Two-dimensional radars are relatively cheap and efficient sensors that often form the first line of defence in airspace control. In military applications, these are often employed as long-range search radars that locate and track aircraft. Depending on the threat evaluation of tracked aircraft, the tracking process is passed along to 3-D search radars or fire control tracking radars once it comes into range of those sensors.

A key component in the above hierarchy is the threat evaluation component. It relies on many factors such as angle of incidence towards the defended assets, time to approach to defended asset, speed of target, and so forth.

In the case of 2-D tracking data a factor that is omitted is the height of the target—as 2-D sensor data does not reflect aircraft altitude. This, however, can be an important consideration, as aircraft altitude limits the attack profiles a target can fly[1].

A relatively small number of papers exist which address the problem of altitude estimation from 2-D radar sources. Each makes use of at least two sensors that are used to trilaterate or triangulate the altitude of the aircraft[2-4]. A related approach was presented by Hakl and le Roux[5] who ignored the altitude of the aircraft and instead described a technique to estimate the vertical activity of an aircraft using a single 2-D radar. This paper presents a method to infer aircraft altitude using a single 2-D radar.

A single 2-D radar source cannot directly determine the altitude of aircraft, thus, the method presented in this paper is coupled with a number of assumptions and limitations.

The terms height and altitude are used interchangeably. Height often refers to the height of an aircraft above ground-level, and altitude, the height of the aircraft above mean sea level. The proposed techniques do not consider terrain, terrain height, or height above mean sea level, but rather the difference in height of the sensor and the observed aircraft.

2. ASSUMPTIONS AND LIMITATIONS

Three limitations are imposed on the method described. These include: (i) aircraft speed is known, (ii) aircraft flies at a level altitude, and (iii) aircraft flies radial towards or away from the 2-D sensor.

Each of these assumptions is relatively soft—if an aircraft violates an assumption only marginally, then the resulting height estimate becomes more inaccurate, but remains computable. Each limitation is described in more depth.

The aircraft speed is instrumental in determining the aircraft altitude—more specifically the horizontal speed of the aircraft. The accuracy to which the speed is known is directly proportional to the accuracy to which the altitude can be determined. Knowledge of aircraft speed can be obtained in a variety of ways. For example, due to the volatile nature of their payload, the speed at which bombers fly is usually controlled by doctrine, similarly, cruise missiles fly at known speeds. On the other hand, many modern 2-D radars can make use of Doppler measurements to determine the radial velocity of the target.

If Doppler measurements are used, then it is important to realise that the speed estimates output by the sensor contain the radial component of the aircraft speed. Since the assumption in this paper is level flight, it should be noted that the Doppler speed measurements underestimate the horizontal speed of the target, which corresponds to an underestimate of the altitude of the target. However, since aircraft are generally further away from the radar horizontally than vertically, the Doppler measurement for

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speed is a sufficiently close approximation for the horizontal speed.

Aircraft are required to fly at a level altitude; and aircraft are required to fly towards or away from the sensor. These two limitations are actually idealizations and are not strictly required. In both the cases it is sufficient to know the horizontal-radial component of the aircraft speed towards the sensor to determine the altitude of the aircraft.

As the results in Section 4 show, the accuracy of aircraft height estimation increases the higher the horizontal-radial speed of the aircraft. If the aircraft is flying perfectly tangential to the radar beam, then the radial speed component is zero and it is impossible to estimate altitude. Conversely, however, it is more accurate to determine the altitude of an aircraft flying at great speeds at a 45 degree angle to the radar beam, than a slow flying aircraft that is flying towards the radar.

To simplify the computational steps, it is assumed that the limitations described above are satisfied.

3. METHODOLOGY USED

In Fig. 1, the circles represent the range spheres of a 2-D radar as seen from the side, thus *a* is higher than *b*. It was observed that the horizontal lines labelled *a* and *b* are of the same length and both touch the outer sphere, however, they both do not end at the same inner sphere.

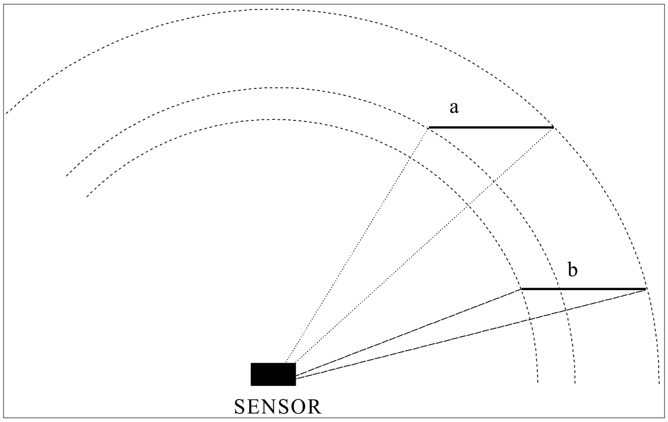

Figure 1. Initial observation.

This observation led to the realisation that it is possible to determine the altitude of aircraft using only a single 2-D radar, provided that the speed is known and the aircraft is flying at a level altitude. Technically, the method described here computes the height of the aircraft above the sensor, but the altitude follows from this given that the altitude of the sensor is known.

The computational process is simple. In Fig. 2, the point marked as t_1 represents the first measurement of the aircraft by the sensor and t_2 represents the second measurement. The r_1 and r_2 correspond to the slant ranges measured by the sensor at t_1 and t_2; and a represents the estimated speed v of the aircraft times the time interval between t_2 and t_1, thus $a = v(t_2 - t_1)$.

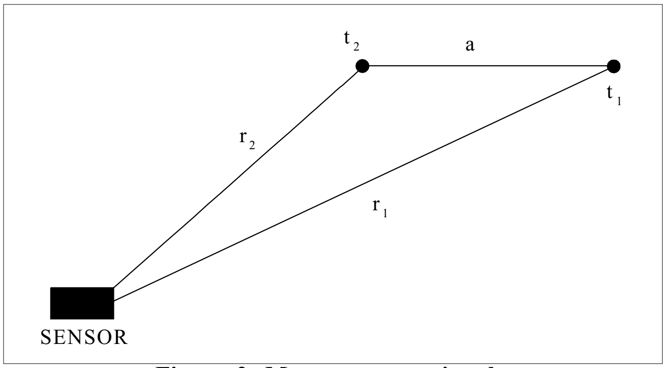

Figure 2. Measurement triangle.

Figure 3 shows the height *h* that one wishes to compute. Since the lengths of the sides of the measurement triangle are known, it is possible to compute the angle $\hat{a}$. Furthermore, noting that due to the assumption of level flight the angle between *a* and *h* is 90°, it follows that $\alpha = 90^\circ - \beta$. Finally the height *h* can be computed noting that $h = c\cos(\alpha)$.

Given the computational triangle shown in Fig. 3, the angle $\hat{a}$ is computed as:

$$\beta = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right),$$

thus, the height *h* is determined as:

$$\begin{aligned} h &= c\cos(\alpha) \\ &= c\cos\left(\frac{\pi}{2} - \beta\right) \\ &= c\cos\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right)\right) \end{aligned}$$

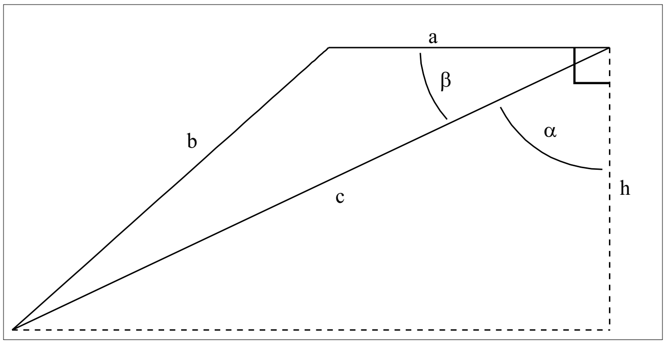

Figure 3. Computational triangle.

4. SENSITIVITY ANALYSIS

Since the values used in this method are the measured values, which are necessarily imprecise, it is instructive to perform a sensitivity analysis to determine the reliability of the values predicted by the method.

To derive a relationship between the estimated height, *h* of the incoming object, and the distance travelled, consider Fig. 4, for the inner circle, which corresponds to the second slant range (distance) measurement.

$$x_1^2 + y_1^2 = r_1^2$$

For the outer circle, which corresponds to the first slant range (distance) measurement:

$$x_2^2 + y_2^2 = r_2^2$$

Using the assumption that the aircraft height remains constant during distance measurements, at a particular y value (where, $y = h$) it is true that:

$$x_2 - x_1 = \sqrt{r_2^2 - h^2} - \sqrt{r_1^2 - h^2}$$

This nonlinear expression leads to the intuitive expectation that the result of the technique will be very sensitive to measurement accuracy. At low altitudes, an extremely small variation in distance measurement implies a significant change in h. However, at very high altitudes, a large variation in the distance measurements implies a very small change in h. A more linear range should emerge between these two extremes, in which the height estimates produced will be useful. To verify this expectation, a sensitivity factor will be derived as follows:

Referring to Fig. 4, the horizontal distance between successive slant range measurements is defined as follows:

$$\Delta x = x_2 - x_1$$

What needs to be determined is how sensitive the value of h is to changes in Δx, bearing in mind the geometrical constraints of Fig. 4. Hence, what must be found is an expression for $\frac{dh}{d\Delta x}$.

From the geometry,

$$\Delta x = \left(r_2^2 - h^2\right)^{\frac{1}{2}} - \left(r_1^2 - h^2\right)^{\frac{1}{2}}.$$

In this expression, h is in fact a function of Δx, whereas for two given measurements, r_1 and r_2 are constant. To highlight this dependency of h on *Äx*, the above expression is written as

$$\Delta x = \left(r_2^2 - h^2(\Delta x)\right)^{\frac{1}{2}} - \left(r_1^2 - h^2(\Delta x)\right)^{\frac{1}{2}}.$$

Now differentiating wrt Δx, on both sides gives

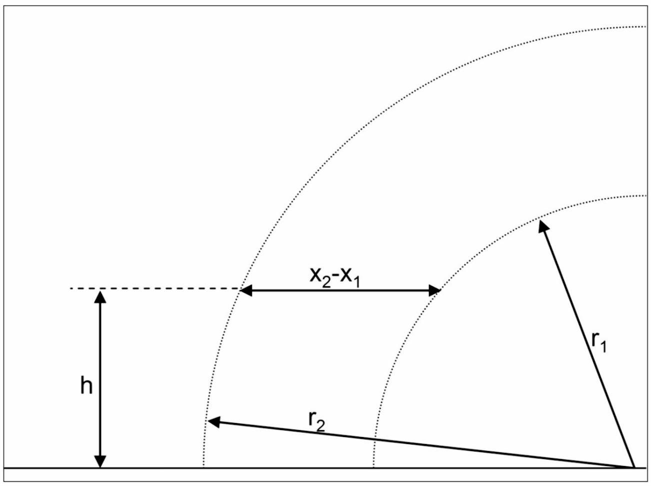

Figure 4. Problem geometry.

$$1 = \left[\frac{1}{2}\left(r_2^2 - h^2(\Delta x)\right)^{-\frac{1}{2}} \cdot -2 \cdot h(\Delta x) \cdot \frac{dh(\Delta x)}{d\Delta x}\right] - \left[\frac{1}{2}\left(r_1^2 - h^2(\Delta x)\right)^{-\frac{1}{2}} \cdot -2 \cdot h(\Delta x) \cdot \frac{dh(\Delta x)}{d\Delta x}\right]$$

$$\frac{dh(\Delta x)}{d\Delta x} = \frac{1}{h(\Delta x) \cdot \left[\frac{1}{\sqrt{\left(r_1^2 - h^2(\Delta x)\right)}} - \frac{1}{\sqrt{\left(r_2^2 - h^2(\Delta x)\right)}}\right]}$$

Hence the desired expression emerges as

$$\frac{dh(\Delta x)}{d\Delta x} = \frac{\sqrt{r_1^2 - h^2(\Delta x)} \cdot \sqrt{r_2^2 - h^2(\Delta x)}}{h(\Delta x) \cdot \left(\sqrt{r_2^2 - h^2(\Delta x)} - \sqrt{r_1^2 - h^2(\Delta x)}\right)}; \; 0 < h < r_1$$

It is clear from this expression that as h->0, $\frac{dh(\Delta x)}{d\Delta x} \to \infty$. This implies that as the incoming aircrafts height h approaches zero, the estimated value of h will become infinitely sensitive to changes in Äx. Likewise, as h->r_1, $\frac{dh(\Delta x)}{d\Delta x} \to 0$. Hence, as the actual height h of the aircraft approaches its range r_1 from the sensor, the estimated aircraft height will become infinitely insensitive to changes in Δx. This result is intuitively satisfying.

5. RESULTS

This section contains preliminary results obtained through simulating the method in software, which varies the parameters within the specified measurement tolerances.

Figures 5 to 7 show error range as a function of increasing horizontal range. Figure 8 to 10 show error range as a function of increasing altitude, while Figs 11 to 13 show error range as a function of increasing speed variance. In Fig. 14, the error range as a function of increasing speed is shown.

The effect of the number of observations on the convergence of the estimate is presented in Figs 15 to 18.

6. DISCUSSIONS

It should be noted that many of the figures presented appear to have unusual behaviour in that they possess abrupt discontinuities. These are caused by a sanitizing computational step that is made during simulation: height estimates below 0 m are treated as height estimates of 0m.

Figures 5 to 7 demonstrate the error range for single measurements as horizontal range from radar to aircraft increases. It is clear that the further away the aircraft is from the radar, the less accurate height estimates become.

Figures 8 to 10 illustrate the error range for single measurements as altitude increases. It is hard to draw

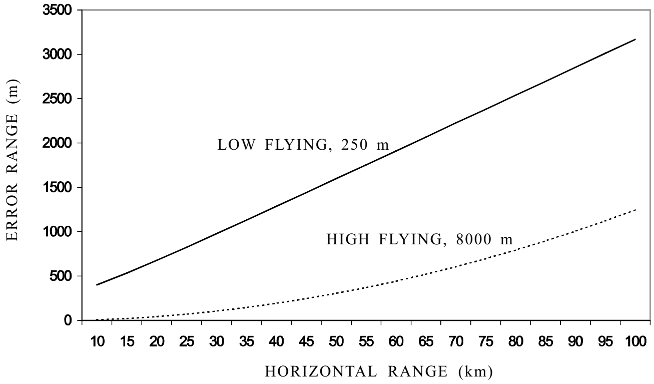

Figure 5. Error range with increasing horizontal range (0.1 m/s speed variance).

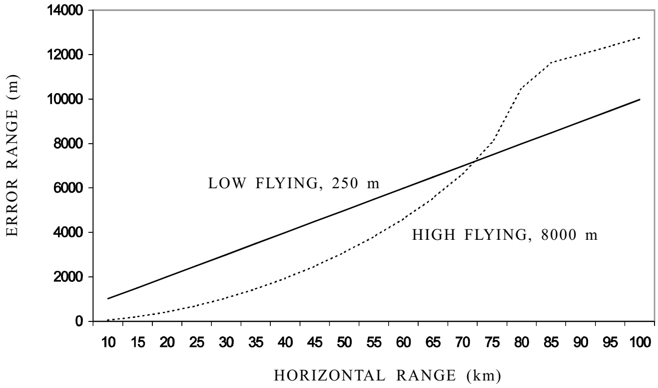

Figure 6. Error range with increasing horizontal range (1 m/s speed variance).

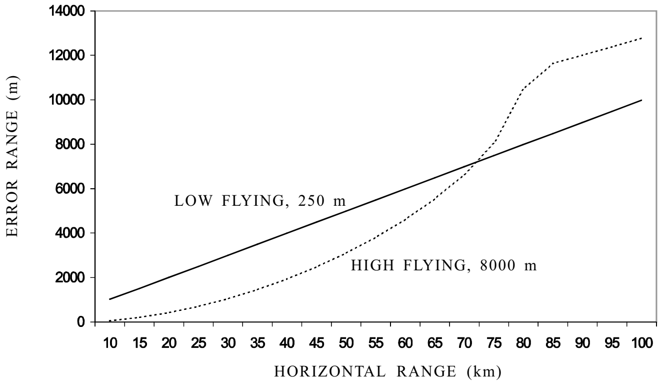

Figure 7. Error range with increasing horizontal range (5 m/s speed variance).

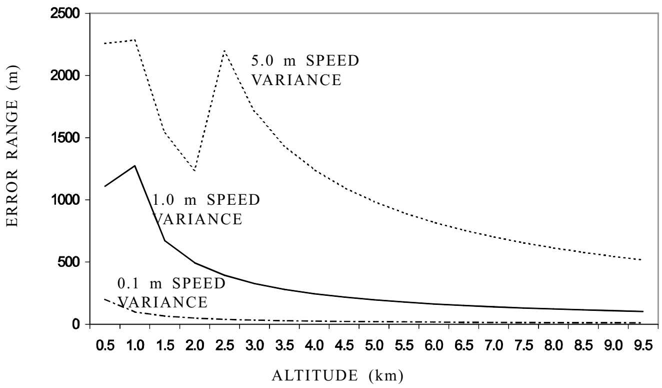

Figure 8. Error range with increasing altitude (10 km horizontal range).

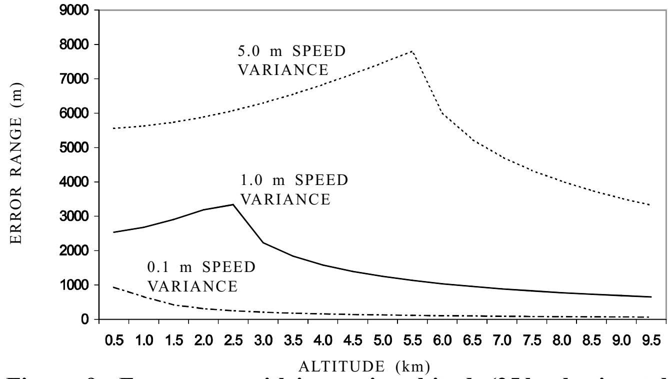

Figure 9. Error range with increasing altitude (25 km horizontal range).

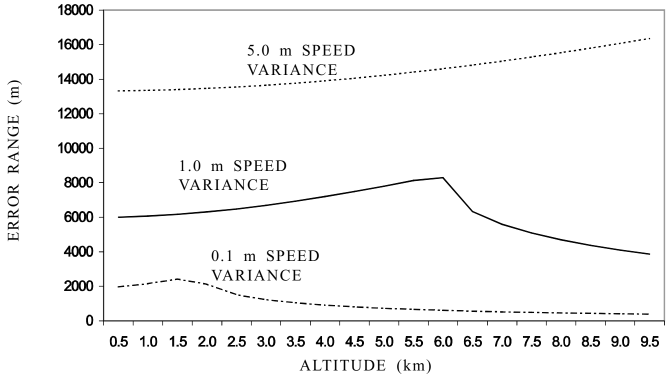

Figure 10. Error range with increasing altitude (60 km horizontal range).

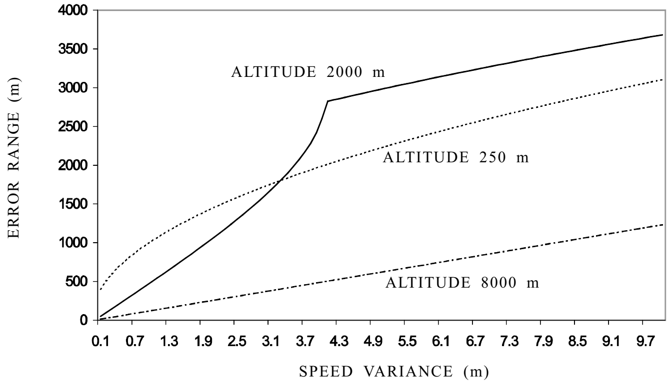

Figure 11. Error range with increasing speed variance (10 km horizontal range).

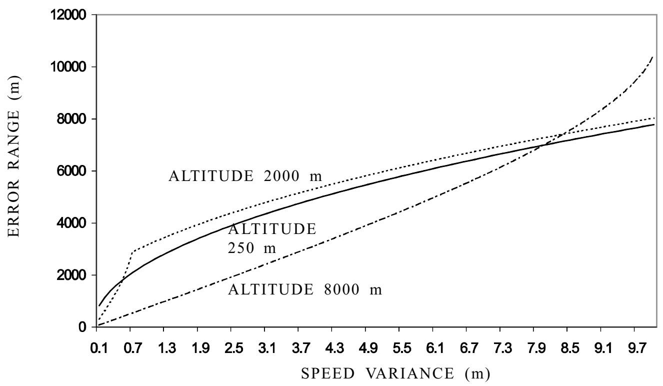

Figure 12. Error range with increasing speed variance (25 km horizontal range).

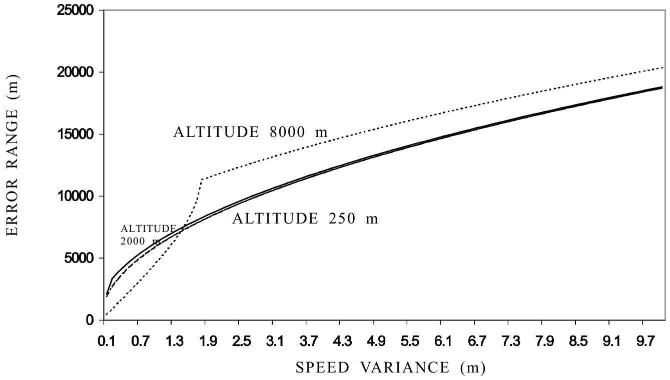

Figure 13. Error range with increasing speed variance (60 km horizontal range).

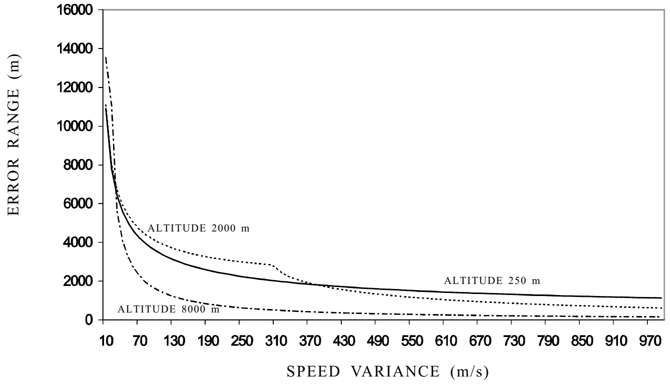

Figure 14. Error range with increasing speed.

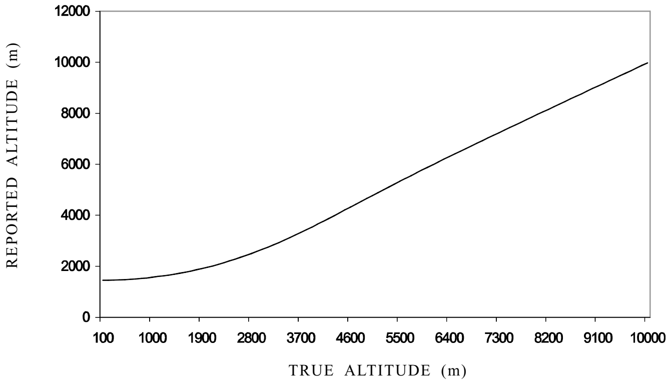

Figure 15. Convergent estimates at 10000 observations (4 m/s speed variance).

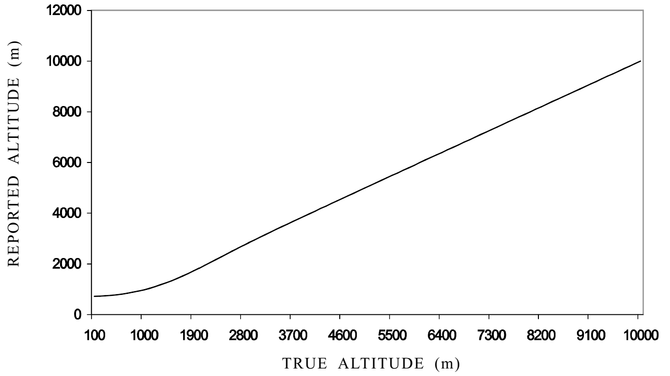

Figure 16. Convergent estimates at 10000 observations (1 m/s speed variance).

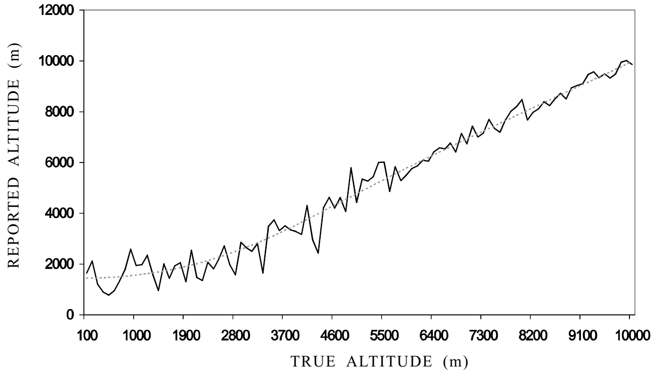

Figure 17. Convergent estimates at 15 observations (4 m/s speed variance).

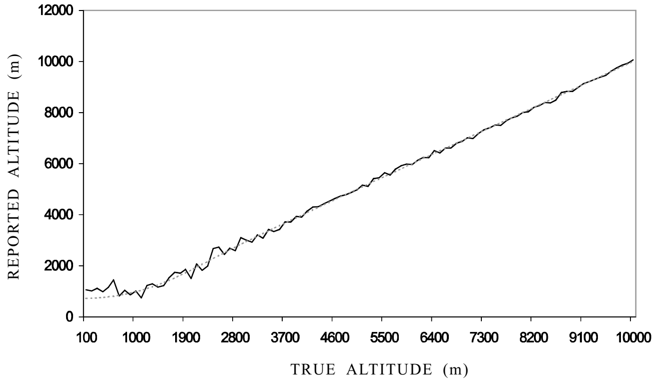

Figure 18. Convergent estimates at 15 observations (1 m/s speed variance).

conclusions regarding trends from these graphs as they indicate error range, not actual error distributions. As seen later, in Figs 17 and 18, an increase in aircraft altitude tends towards a more accurate estimate of aircraft altitude.

Figures 11 to 13 indicate the error range of single measurements as the variance in speed measurements increases. It is clear that the error increases as the variance in speed increases.

Figures 14 depicts the error range as the speed of the target increases. It is clear that the faster the aircraft flies, the more accurate a height estimate is found using the methods described in this paper.

Naturally, as more observations are made, the height estimates become progressively more accurate. Figures 15 and 16 demonstrate that with sufficient observations a clear one-to-one mapping is possible between the reported and the true altitudes.

Figures 17 and 18 are of particular interest. These represent the expected accuracy in height estimation (as simulated) after 15 observations. At an update rate of 4 s per observation, this is indicative of the accuracy to which aircraft height is estimated after 1 min of observation of the aircraft. Note that both have a lower limit of reported altitude which intuitively corresponds to the sensitivity analysis described earlier in this paper.

It is important to realise that radar sensitivities are not static, instead these are subject to the quality of the processing of the received signals as well as configuration of the radar itself. A radar's parameters can be optimised to achieve higher sensitivities towards range estimates, overall range, speed estimates, and so forth.

The results displayed in Fig. 17 are the results with a variance of 4 m/s to the speed estimate of the aircraft. This is a value chosen to be plausible in modern 2-D radars and achievable purely through upgrades to the software and signal processing as well as configuring the parameters of the radar towards improved speed sensitivity.

As Fig. 17 indicates, the simulation suggests that (at the indicated speed accuracy) height estimates of aircraft after 15 measurements are accurate to within 1000 m. This is sufficient to clearly identify an aircraft as flying a low or a high profile. Albeit at that accuracy, the method has difficulty distinguishing between aircraft flying at altitudes of 0 m to 3000 m.

Figure 18, in contrast, represents results at a speed variance of 1m/s. This is indicative of the potential performance of 2-D radars designed and optimized towards the estimation of height of aircraft using the methods described in this paper. The simulation suggests that at this speed accuracy the height estimates for aircraft is accurate to within 100 m; though low-flying aircraft between 0 m and 1500 m are hard to be distinguished.

7. FUTURE WORK

Future work includes incorporating sensor error models into the height estimation. This will also aid in the comparison of real world data results with simulated results, as real world data inescapably include sensor errors. An experiment is being planned for this purpose. Sensor errors that will most likely influence the height estimation accuracy are:

- Range measurements: Range errors will have a definite effect on the height estimation. Refer to the equation for $\frac{dh}{d\Delta x}$ which is dependant on the ranges r_1 and r_2.
- Bearing (direction) measurements: When calculating the horizontal radial component of the aircraft velocity, incorrect bearing measurements may have an influence on the accuracy of the radial component, which in turn will influence the height estimation.
- Doppler measurements: Accurate Doppler measurements result in accurate radial velocity component estimations.
- Time measurements: Timing of measurements has a direct influence on accuracy.

8. CONCLUSIONS

This paper presents a method to infer aircraft altitude using a single 2-D radar. Simulation results based on the method were presented, which show that provided the motion of the aircraft complies with three loose constraints and the aircraft, it does not fly very high or very low, useful estimates of the height are obtained.

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